

# INTERNATIONAL COOPERATION IN POLLUTION CONTROL\*

**Santiago J. Rubio and Begoña Casino\*\***

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Correspondence to Santiago J. Rubio, University of Valencia, Dpto. de Análisis Económico, Campus de los Naranjos s/n, Edificio Departamental Oriental, 46022 Valencia, Spain. E-Mail: Santiago.Rubio@uv.es

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\*\* S.J. Rubio and B. Casino: University of Valencia.

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## **ABSTRACT**

In this paper the profitability and stability of an International Environmental Agreement among  $N$  identical countries that emit a pollutant are studied using a standard quadratic net benefit function. The static analysis shows that only a bilateral agreement could be self-enforcing independently of the number of countries affected by the externality and the gains coming from cooperation. It is also shown that this result occurs both when the coalition takes as given the emissions of nonsignatories and when it acts as the leader of the game. In the second part of the paper a differential game is proposed in order to analyze the stock externality due to accumulated emissions. Similar results to the ones obtained for the static model are derived both for an open-loop Nash equilibrium and for a feedback Nash equilibrium in linear strategies.

**KEYWORDS:** International Environmental Agreements; Flow and Stock Externalities; Differential Games; Open-Loop Nash Equilibrium; Feedback Nash Equilibrium; Linear Strategies.

# 1 INTRODUCTION

The increasing social and political interest in global environmental problems is only one of the aspects of the increasing interdependence among countries in recent years. Global warming, depletion of the ozone layer and loss of biological diversity are examples of environmental problems related with global commons that require policy coordination. The Framework Convention on Climate Change, the Montreal Protocol on Substances that Deplete the Ozone Layer and the Convention on Biodiversity are the most important *International Environmental Agreements* (IEA) signed to date. There are two main issues related to international environmental cooperation: the profitability and the stability of the agreements. The profitability refers to the potential gains coming from the cooperation among countries when reciprocal negative externalities exist. The question regarding the stability of an agreement arises due to the absence of an international authority or an international law that compels countries to take part in or respect the agreement. Thus, countries may face a prisoner's dilemma whereby, although there are greater benefits to be gained through full cooperation, each one has incentives to unilaterally defect from the agreement.<sup>1</sup>

Different papers have been published in recent years on these issues. Among them we would like to highlight the ones written by Heal (1992), Hoel (1992), Carraro and Siniscalco (1993), Barrett (1994, 1997b), Sandler and Sargent (1995) and Na and Shin (1998). In all these papers a noncooperative game-theory analysis of coalition formation among two or more countries is presented assuming that environmental damages are associated with *flow* externalities and that all the players move only once.<sup>2</sup> From these papers we are interested in the results obtained by Hoel (1992) and Barrett (1994). In these two papers the number of signatories, the terms of the self-enforcing agreement and the actions of nonsignatories are all determined endogenously using *numerical* examples. Hoel finds for a model of *constant* marginal environmental cost that only two countries cooperate in equilibrium

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<sup>1</sup>Good surveys on global environmental problems and international environmental agreements are Barrett (1997a), Carraro and Siniscalco (1998), Swanson and Johnston (1999) and Finus (2000).

<sup>2</sup>For a cooperative approach see Chander and Tulkens (1992, 1997) and van Egteren and Tang (1997). In Barrett (1994) and Finus and Rundshagen (1998) the stability of the agreement is studied in the framework of an infinitely repeated game applying the concept of renegotiation-proofness.

almost independently of the total number of countries, and that this equilibrium is for all practical purposes equal to the noncooperative equilibrium. In his model it is assumed that the agreement among countries with a different marginal environmental cost consists of a uniform percentage reduction of their emissions which is determined by the most preferred reduction for the median country (among the signatories countries).<sup>3</sup> In Barrett (1994) it is assumed that all the countries are identical and that the signatories act as the leader of the game, i.e. they choose their level of abatement to maximize their collective net benefits subject to the reaction functions of nonsignatories. His results show for the case of *linear* marginal abatement benefits and costs that, when cooperation can increase net benefits substantially, the self-enforcing IEA cannot sustain a large number of signatories. However, when the gains from cooperation are low, a lot of countries would be interested in signing the agreement. Finally, we want to mention that the profitability of the cooperation among countries suffering a negative *stock* externality has been analyzed by Long (1992), van der Ploeg and de Zeeuw (1992), Dockner and Long (1993), Xepapadeas (1995) and Dockner and Nishimura (1999).<sup>4</sup> In these papers, the interdependence among countries is modeled as a differential game or as a difference game, and the noncooperative outcome as a feedback Nash equilibrium. The study developed by these authors focuses on the comparison between the efficient solution and the noncooperative solution of the game but does not address the analysis of the stability of the agreements.<sup>5</sup>

Our paper has two parts. In the first part, we present a full characterization of the self-enforcing agreement obtained *analytically* for a model of

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<sup>3</sup>Petrakis and Xepapadeas (1996) extends, using also a model with constant marginal environmental cost, Carraro and Siniscalco's (1993) results to the case in which the participating countries are not identical. They show that if a group of countries commits to cooperation, there exists a system of self-financed side payments, such that the rest of countries involved in the environmental externality enter the coalition and reduce emissions.

<sup>4</sup>See Kaitala, Pohjola and Tahvonen (1992), Tahvonen (1994), Escapa and Gutiérrez (1997) and Mäler and de Zeeuw (1998) for simulations, in a dynamic framework, on how the potential gains derived from cooperation would be distributed among countries. Tahvonen (1994) and Escapa and Gutiérrez (1997) focus on the global warming problem whereas Kaitala, Pohjola and Tahvonen (1992) and Mäler and de Zeeuw (1998) address the acid rain problem.

<sup>5</sup>In Dockner and Nishimura (1999) the dynamics of the feedback Nash equilibrium for the case of one-sided transboundary pollution is analyzed.

emission control with a standard quadratic net benefit function and identical countries. We also study the importance of the equilibrium concept used to solve the game considering, firstly, that the signatories take as given the emissions of the nonsignatories countries, and, secondly, that the coalition of signatories behaves as a leader. Our results show that in both cases the unique self-enforcing IEA consists only of two countries independently of the scope of the gains and the number of the countries involved in the externality. This result generalizes for a model with linear marginal environmental costs the findings obtained by Hoel (1992). Moreover, our analysis shows that the model of emissions used in our paper is not completely symmetric to the model of abatement developed by Barrett as long as we obtain different conclusions.<sup>6</sup> We find that the net benefits of signatories and nonsignatories increase with cooperation. However, the incentive for a country to act as a free rider is big enough as if to prevent cooperation once a coalition of two countries has been reached. In the second part of the paper we extend the analysis of the agreement stability to the case of a stock externality, thus advancing the analysis developed in the papers just quoted. Following van der Ploeg and de Zeeuw (1992) and Dockner and Long (1993), we develop an international pollution control model with  $N$  identical countries where the interactions between signatory and nonsignatory countries are represented by a differential game. In a first approximation of the problem of stability using open-loop strategies, we find that, for any size of coalition, countries get smaller payoffs if they cooperate. A numerical example confirms that, in a dynamic framework, the scope of the agreement is also limited. Our results show that a *bilateral* coalition is the unique self-enforcing IEA independently of the gains coming from cooperation. In the last part of the paper, we calculate the feedback Nash equilibrium in linear strategies for the same numerical example and we find again the same result. The sensitivity analysis shows that this result is robust. The intuition is that agents do not find it profitable to select punishment strategies if these are not credible. In other words, if they have a negative effect on their own payoffs higher than the negative effect of accommodating to the exit of one of the countries in

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<sup>6</sup>The difference in the results is explained by the different modelling of the environmental externalities. Thus, whereas Barrett uses a model of *abatement* where the interdependence among countries occurs through the benefit function, we work with a model of *emissions* where the interdependence passes through the environmental damage function and this difference is relevant in the analysis of the stability of the agreements as our results show.

the agreement. Then, given that the incentive to act as a free rider is sufficiently large and that the payoffs of the signatories increase with respect to the number of countries belonging to the agreement, the result is that only a bilateral agreement can be self-enforcing.

In Section 2 the static model is presented and the efficient equilibrium and the Cournot equilibrium are calculated. In Section 3 the stability analysis is developed, assuming firstly, Subsection 3.1, that the signatories countries take as given the emissions of the nonsignatory countries, to study, secondly, Subsection 3.2, the Stackelberg equilibrium. The dynamic model is studied in Section 4. In this Section the efficient equilibrium, the open-loop Nash equilibrium and the feedback Nash equilibrium are calculated and compared. In Section 5 the stability of an IEA is analyzed using a numerical example both for an open-loop Nash equilibrium, Subsection 5.2, and for a feedback Nash equilibrium, Subsection 5.4. Some concluding remarks end the paper.

## 2 THE STATIC MODEL

A pollutant is emitted by  $N$  identical countries that share a natural resource as the environment. Define  $q_i$  as emissions by country  $i$ . These emissions are associated with some natural resource, say oil, whose consumption provides, directly or indirectly, some utility. Therefore, each country gets some benefits from its emissions and suffers a damage due to aggregate emissions,  $Q = \sum_i q_i$ .

Let's assume a quadratic benefit function for each country

$$B_i(q_i) = aq_i - \frac{b}{2}q_i^2, \quad a > 0, \quad b > 0, \quad (1)$$

a quadratic damage function

$$C_i = \frac{c}{2}Q^2, \quad c > 0, \quad (2)$$

and denote country  $i$ 's net benefits by

$$\pi_i = aq_i - \frac{b}{2}q_i^2 - \frac{c}{2}Q^2.$$

## 2.1 Full-cooperation versus non-cooperation

The gains coming from full-cooperation will be given by the difference between net benefits under cooperation and net benefits got by countries when they do not cooperate. The level of emissions that maximizes aggregate net benefits  $\Pi = \sum_i \pi_i$  is found by setting each country's marginal benefits of emissions equal to global marginal costs. Thus the full cooperative level of emissions,  $q_i^*$ , and net benefits,  $\pi_i^*$ , for each country are given by

$$q_i^* = \frac{a}{b + N^2 c}, \quad \pi_i^* = \frac{a^2}{2(b + N^2 c)}. \quad (3)$$

The noncooperative outcome arises when each country chooses its level of emissions taking as given the level of emissions from all the other countries. The optimal solution consists of setting each country's marginal benefit of emissions equal to its own marginal cost. This Cournot equilibrium is given by the following level of emissions,  $q_i^c$ , and net benefits,  $\pi_i^c$ :

$$q_i^c = \frac{a}{b + Nc}, \quad \pi_i^c = \frac{a^2 [b - (N - 2) Nc]}{2(b + Nc)^2}. \quad (4)$$

Since we are analyzing a model of partial equilibrium, it is possible to find some substitute for the resource so that this will be exploited only if net benefits are strictly positive. This occurs when  $b/c > N(N - 2)$ . That is, when the rate of decrease of marginal benefits relative to the rate of increase of marginal costs is enough large. We assume in the rest of the paper that this inequality is satisfied.

The gains to cooperation for each country are given by the following expression:

$$\frac{\pi_i^* - \pi_i^c}{\pi_i^c} = \frac{(N - 1)^2 N^2}{(b/c + N^2) [b/c - (N - 2) N]}.$$

Thus, full-cooperation is more profitable when  $b/c$  takes a small value and we can state the following:

**Proposition 1** *The gains to cooperation depend positively on the slope of the marginal cost function and negatively on (the absolute value of) the slope of the marginal benefit function.*

Nevertheless, the optimality of full-cooperation is not enough to guarantee a stable coalition including the  $N$  countries because each one may have incentives to unilaterally defect from the agreement. For this reason, it is also necessary to study the stability of a coalition.

### 3 STABILITY OF AN INTERNATIONAL ENVIRONMENTAL AGREEMENT

Suppose that  $n$  countries negotiate an IEA and the other  $N - n$  countries decide to be outside the coalition. The number of signatories that sustains a stable agreement will be obtained by using the concept of stability developed by d'Aspremont et al. (1983) for the analysis of a cartel. This stability concept has already been used by Hoel (1992) and Barrett (1994) to analyze the self-enforcement of an IEA. Let  $\pi_j$  be net benefits of a country  $j$  that does not belong to the coalition of countries that sign the IEA and  $\pi_i$  net benefits of a signatory country. According to Barrett (1994) a self-enforcing agreement can be defined as follows.

**Definition 1** *An IEA consisting of  $n$  signatories is self-enforcing if  $\pi_i(n) \geq \pi_j(n-1)$  and  $\pi_j(n) \geq \pi_i(n+1)$ , where  $i = 1, \dots, n$  and  $j = 1, \dots, N - n$ .*

The first inequality holds if signatory countries have no incentives to withdraw from the coalition because the increase in the costs due to the increase in aggregate emissions would be higher than the benefit provided by an increase in their emissions. The second inequality requires that nonsignatories do not want to accede to the coalition because the decrease in the costs due to a reduction in aggregate emissions would be smaller than the decrease in their benefits resulting from the reduction of their emissions.

Net benefits of a nonsignatory country  $j$  are given by the following expression

$$\pi_j = aq_j - \frac{b}{2}q_j^2 - \frac{c}{2} \left( \sum_{i=1}^n q_i + \sum_{j=1}^{N-n} q_j \right)^2.$$

The level of emissions for which marginal benefits are equal to marginal costs provides the maximum benefit of each country  $j$ . Under the assumption of symmetry this condition is:



$$a - bq_j = c[nq_i + (N - n)q_j], \quad j = 1, \dots, N - n. \quad (5)$$

This equation implicitly defines the reaction function for the  $N - n$  countries that are outside the coalition. Thus, the optimal level of  $q_j$  depends on emissions by signatories.

### 3.1 Cournot conjecture

Signatories are assumed to coordinate for the same level of emissions in order to maximize their collective net benefits taking as given the emissions of nonsignatories. Then, the reaction functions of countries belonging to the coalition are given by the following expression

$$a - bq_i = nc[nq_i + (N - n)q_j], \quad i = 1, \dots, n. \quad (6)$$

The intersection between the two best reply functions defined by Eqs. (5) and (6) determines the optimal levels of emissions

$$q_i^c = \frac{a[b - (N - n)(n - 1)c]}{b[b + (N + n^2 - n)c]}, \quad i = 1, \dots, n, \quad (7)$$

$$q_j^c = \frac{a[b + n(n - 1)c]}{b[b + (N + n^2 - n)c]}, \quad j = 1, \dots, N - n, \quad (8)$$

and, then, aggregate emissions are

$$Q^c = \frac{Na}{b + (N + n^2 - n)c}. \quad (9)$$

It is easy to verify that the full cooperative ( $n = N$ ) and noncooperative solutions ( $n = 0$  or  $n = 1$ ), given by Eqs. (3) and (4), are special cases of Eqs. (7) and (8).

Notice that  $q_i^c$  could take negative values. However, it is immediate to check that for  $b/c > N(N - 2)$  the expression (7) is positive for all  $n \in$

$[2, N-1]$ .<sup>7</sup> Moreover, aggregate emissions decrease as the size of the coalition increases and, if we take the derivative of the expressions (7) and (8) with respect to  $n$ , we have, respectively,

$$\frac{\partial q_j^c}{\partial n} = \frac{(2n-1) N c^2 a}{b [b + (N + n^2 - n) c]^2} > 0,$$

$$\frac{\partial q_i^c}{\partial n} = \frac{[(n^2 - N) c - b] N c a}{b [b + (N + n^2 - n) c]^2}.$$

Thus, emissions by nonsignatory countries increase with the size of the coalition and the condition  $b/c > N(N-2)$  guarantees that emissions by signatories decrease.<sup>8</sup>

Comparing the level of emissions given by Eqs. (7) and (8) we get

$$q_i^c - q_j^c = -\frac{(n-1) N c a}{b [b + (N + n^2 - n) c]} < 0 \text{ for all } n \geq 2.$$

Therefore, signatory countries emit less pollutants than nonsignatories. Since the benefit and cost functions are identical for both types of countries and the costs depend on aggregate emissions, a country that emits more than other has higher net benefits. Consequently, we can conclude that  $\pi_i^c(n) < \pi_j^c(n)$  for  $n \in [2, N-1]$ . Moreover, it is easy to show that both net benefits are increasing with respect to the number of signatories. Using (7) and (8) we can write the net benefits as

$$\pi_i^c(n) = \frac{a^2}{2b} - \frac{N^2 c a^2 (b + n^2 c)}{2b [b + (N + n^2 - n) c]^2}, \quad (10)$$

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<sup>7</sup>The optimal level of emissions of signatories is positive for  $b/c > (N-n)(n-1)$  but as  $N(N-2) > (N-n)(n-1)$  for  $n \in [2, N-1]$ , we have that, if the condition for positive net benefits,  $b/c > N(N-2)$ , is satisfied,  $b/c > (N-n)(n-1)$  is also satisfied and  $q_i^c$  is positive.

<sup>8</sup>The partial derivative  $\partial q_i^c / \partial n$  is negative for  $b/c > n^2 - N$ . From  $b/c > (N-2)N$  we have that  $(n^2 - N) - b/c < (n^2 - N) - (N-2)N = n^2 - N(N-1)$ . This expression increases with  $n$  and takes a negative value for  $n = N-1$ . Consequently,  $(n^2 - N) - b/c < 0$  for all  $n \in [2, N-1]$ , and  $\partial q_i^c / \partial n$  must be negative in this case.

$$\pi_j^c(n) = \frac{a^2}{2b} - \frac{N^2 c a^2 (b+c)}{2b [b + (N + n^2 - n) c]^2}. \quad (11)$$

It is immediate from this last expression that the net benefits of nonsignatories increase as the number of countries that sign the agreement increases. For the signatories, the derivative of net benefits with respect to  $n$  is

$$\frac{\partial \pi_i^c}{\partial n} = - \frac{4a^2 c^2 b N^2 [b + (N + n^2 - n) c] [cn(N - n^2) - b(n - 1)]}{4b^2 [b + (N + n^2 - n) c]^4}$$

where  $cn(N - n^2) - b(n - 1)$  is negative if  $b/c > N(N - 2)$ . In this case, the net benefits of signatories also increase with the number of countries in the agreement.

However, as our definition of stability depends on the comparison between  $\pi_i^c(n)$  and  $\pi_j^c(n - 1)$ , we start at  $n = 2$  comparing  $\pi_i^c(2)$  with  $\pi_j^c(1)$  in order to solve for the stable number of signatories. This difference is given by the following expression

$$\pi_i^c(2) - \pi_j^c(1) = \frac{N^2 c^2 a^2 [b^2 - 2(N - 4)bc - (3N^2 - 4N - 4)c^2]}{2b [b + (N + 2)c]^2 [b + Nc]^2}.$$

The sign of this difference depends on the sign of the expression into square brackets in the numerator. It is easy to show that this expression is positive for  $b/c > N(N - 2)$  so that we can conclude that  $\pi_i^c(2) > \pi_j^c(1)$  for this case.<sup>9</sup> This means that at least two countries could improve by cooperating. Now, according to our definition of stability, we have to check if there are incentives for a nonsignatory country to cooperate with this coalition. Thus, if we compare  $\pi_j^c(2)$  with  $\pi_i^c(3)$  we have that

$$\pi_j^c(2) - \pi_i^c(3) = \frac{4N^2 c^3 a^2 [(N - 1)b + N(N + 3)c]}{b [b + (N + 2)c]^2 [b + (N + 6)c]^2} > 0.$$

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<sup>9</sup>The expression into square brackets is positive for  $b/c > N - 4 + 2(N^2 - 3N + 3)^{1/2}$ . Let's suppose that  $N(N - 2) < N - 4 + 2(N^2 - 3N + 3)^{1/2}$  which implies that  $N^2 - 3N + 4 < 2(N^2 - 3N + 3)^{1/2}$ . Then the square on the left-hand side must be smaller than the square on the right-hand side which yields the following contradiction:  $N^2(N - 3)^2 + 4N(N - 3) + 4 < 0$  for  $N \geq 2$ , and we can write that  $b/c > N(N - 2) > N - 4 + 2(N^2 - 3N + 3)^{1/2}$  and conclude that the expression into brackets is positive.

Therefore, a bilateral coalition satisfies the stability condition stated in Definition 1 because  $\pi_i^c(2) > \pi_j^c(1)$  and  $\pi_j^c(2) > \pi_i^c(3)$ . On the contrary, a coalition of three countries violates the inequality  $\pi_i^c(n) \geq \pi_j^c(n-1)$ .

In general, from Eqs. (10) and (11) we can get the difference between  $\pi_j^c(n-1)$  and  $\pi_i^c(n)$  for any size of the coalition as

$$\pi_j^c(n-1) - \pi_i^c(n) = \frac{(n-1)N^2c^2a^2[(n-3)b^2 + Abc + Bc^2]}{2b[b + (N + (n-1)(n-2))c]^2[b + (N + n^2 - n)c]^2},$$

$$A \equiv 2[(n-1)N + n^3 - 4n^2 + 3n - 2],$$

$$B \equiv (n+1)N^2 + 2n(n^2 - 2n - 1)N + n^2(n-1)^2(n-3),$$

where  $A$  and  $B$  are positive for  $n \geq 3$ . Thus, if we start at  $n = N$  we find that the incentives to form a coalition arise only for  $n = 2$ . Therefore, we can conclude the following:

**Proposition 2** *A Cournot-IEA consisting of two countries is the unique self-enforcing IEA, independently of the scope of the gains to full-cooperation and the number of countries.*

This results can be better understood by considering an example. Table 1 shows the net benefit and emissions corresponding to each possible agreement, and Fig. 1 illustrates the example.

To solve for  $n^*$ , start at  $n = 1$  and compare  $\pi_j(1)$  with  $\pi_i(2)$ . Clearly,  $3306 < 3418$ , and hence it will play a nonsignatory to enter into the agreement. Now compare  $\pi_j(2)$  with  $\pi_i(3)$ . In this case,  $3897 > 3799$ , and it will not pay a nonsignatory to cooperate. Likewise, starting at  $n = 10$ , one finds that signatories always do better by withdrawing from the agreement whenever  $n > 2$ . Hence, a bilateral agreement is the only self-enforcing IEA. In Fig. 1 we represent the net benefits obtained by a country that belongs to a coalition of size  $n$ ,  $\pi_i^c(n)$  and the net benefits that the country could get by leaving the coalition,  $\pi_j^c(n-1)$ . These two functions intersect each other between  $n = 2$  and  $n = 3$ , so that a nonsignatory has no incentives to join a bilateral coalition. Similarly, starting at  $n = 10$ , signatories have an strong incentive to unilaterally defect from the great coalition and this happens for every coalition greater than two.

**Table 1: Stability analysis for the numerical example. Static model.\***

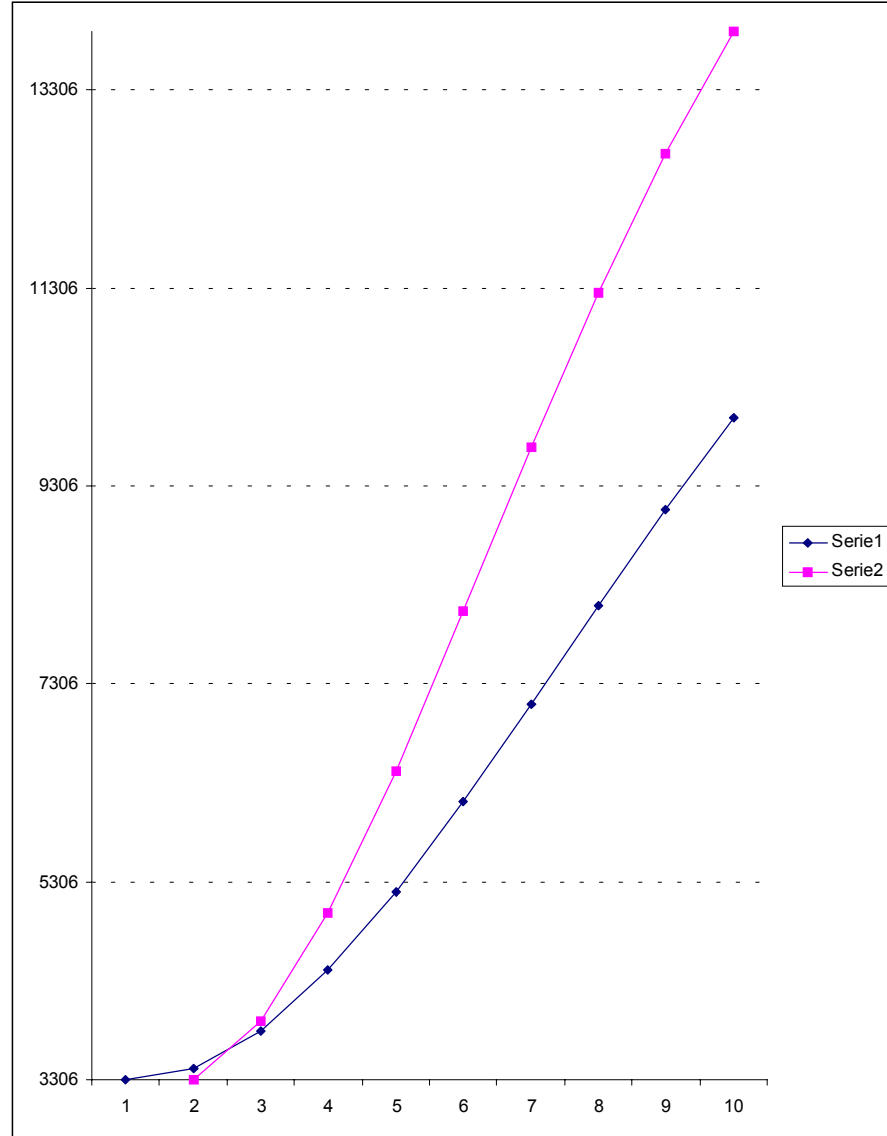
n	$q_i$	$q_j$	Q	$\pi_i$	$\pi_j$	$\Pi$
1		36.3636	363.636		3306	33060
2	32.8571	36.4286	357.143	3418	3897	38012
3	29.6552	36.5517	344.828	3799	4988	46313
4	26.8852	36.7213	327.869	4413	6428	56220
5	24.6154	36.9231	307.692	5207	8047	66270
6	22.8571	37.1429	285.714	6122	9694	75508
7	21.5789	37.3684	263.158	7102	11257	83485
8	20.7229	37.5904	240.964	8097	12669	90114
9	20.2198	37.8022	219.780	9071	13902	95541
10	20.0000		200.000	10000		100000

\*Assumes N=10, a=1000, b=25 and c=0.25

The agreement involving two countries only causes a small reduction in aggregate emissions of about a 1.8 % and an increase in aggregate net benefits of 15 %. These gains are small regarding the gains to full cooperation that represents a 202.5%. In other words, this equilibrium is for all practical purposes almost identical to the noncooperative equilibrium.

Next, in order to see the implications of the concept of equilibrium used to solve the game, we analyze the Stackelberg equilibrium.

**Figure 1: Stable coalition under Cournot conjecture.**



$N=10$ ,  $a=1000$ ,  $b=25$  and  $c=0.25$

Serie 1:  $\pi_i(n) \equiv$  net benefits of signatory countries for a coalition of size  $n$ .

Serie 2:  $\pi_j(n-1) \equiv$  net benefits of nonsignatory countries for a coalition of size  $n-1$ .

### 3.2 Leadership

Let's now assume that the coalition acts as a leader. From (5) we get the reaction function of nonsignatories

$$q_j = \frac{a - ncq_i}{b + (N - n)c}. \quad (12)$$

The countries that belong to the coalition choose  $q_i$  to maximize their collective net benefits, with  $q_i$  being identical for all of them, subject to (12). The solution is

$$q_i^s = \frac{[(b + (N - n)c)^2 - (N - n)nc]a}{b[(b + (N - n)c)^2 + n^2bc]}, \quad i = 1, \dots, n. \quad (13)$$

Substituting into (12) yields

$$q_j^s = \frac{[(b - nc)(b + (N - n)c) + n^2bc]a}{b[(b + (N - n)c)^2 + n^2bc]}, \quad j = 1, \dots, N - n. \quad (14)$$

Accordingly, aggregated emissions are

$$Q^s = \frac{[b + (N - n)c]Na}{(b + (N - n)c)^2 + n^2bc}.$$

Notice that the noncooperative Stackelberg equilibrium is given by Eqs. (13) and (14) for  $n = 1$  and the noncooperative Cournot equilibrium by Eq. (14) for  $n = 0$ .

It is also possible to prove (see Appendix A) that, as happened under Cournot conjecture, if the condition for positive net benefits is satisfied, emissions by signatory countries decrease with the size of the coalition while emissions by nonsignatories increase.

$$\frac{\partial q_i^s}{\partial n} = \frac{[n^2(b + c)c - (b + Nc)^2]Nca}{(b + (N - n)c)^2 + n^2bc} < 0, \quad (15)$$

$$\frac{\partial q_j^s}{\partial n} = - \frac{[n^2 (b+c) c - 2n (b+Nc) (b+c) + (b+Nc)^2] Nca}{b [(b+(N-n)c)^2 + n^2 bc]} > 0. \quad (16)$$

The effect on aggregate emissions is given by the following expression that takes a negative value.<sup>10</sup>

$$\frac{\partial Q^s}{\partial n} = \frac{[n^2 (b+c) c - 2n (b+Nc) (b+c) + (b+Nc)^2] Nca}{[(b+(N-n)c)^2 + n^2 bc]^2} < 0.$$

That is, the increase of emissions by nonsignatory countries is compensated by the decrease of emissions by signatories. If we compare the emissions of both types of countries we have

$$q_i^s - q_j^s = \frac{[(N-n)c - (n-1)b] Nca}{b [(b+(N-n)c)^2 + n^2 bc]}.$$

>From this difference a critical value for  $n$  is obtained:  $n^+ = (N + b/c) / (1 + b/c)$ , so that if  $n < n^+$  then  $q_j^s < q_i^s$  and consequently  $\pi_j^s(n) < \pi_i^s(n)$  and if  $n > n^+$  then  $q_j^s > q_i^s$  and  $\pi_j^s(n) > \pi_i^s(n)$ . Thus, if condition  $b/c > N(N-2)$  is satisfied,  $n^+$  presents an upper bound equal to  $N/(N-1)$ . As this value is lower than two, we have that  $1 < n^+ < 2$  for any value of  $b/c$  higher than  $(N-2)N$ , and then  $\pi_i^s(1) > \pi_j^s(1)$  whereas  $\pi_i^s(n) < \pi_j^s(n)$  for  $n \geq 2$ . Using (13) and (14) we can write the net benefit functions as

$$\pi_i^s(n) = \frac{a^2}{2b} - \frac{N^2 ca^2}{2 [(b+(N-n)c)^2 + n^2 bc]},$$

$$\pi_j^s(n) = \frac{a^2}{2b} - \frac{(b+c) [b+(N-n)c]^2 N^2 ca^2}{2b [(b+(N-n)c)^2 + n^2 bc]^2},$$

which, as it occurred under Cournot conjecture, are increasing with respect to the number of countries that join in the agreement.<sup>11</sup>

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<sup>10</sup>Notice that the numerator in  $\partial Q^s / \partial n$  is equal to the numerator in (16).

<sup>11</sup>Again the only condition we need to obtain this result is that  $b/c > N(N-2)$ .



Next, we investigate whether the unique self-enforcing IEA is also, for the leadership case, a coalition consisting of two countries. For  $n = 2$ , we get

$$\pi_i^s(2) - \pi_j^s(1) = \frac{N^2 a^2 [b^4 c^2 + 2Nb^3 c^3 + Cb^2 c^4 + Dbc^5 + Ec^6]}{2b [(b + (N-1)c)^2 + bc]^2 [(b + (N-2)c)^2 + 4bc]},$$

where

$$\begin{aligned} C &\equiv 2N^2 - 1, \quad D \equiv (N-1)(2N^2 - 5N + 5), \\ E &\equiv (N-2)^2(N-1)^2. \end{aligned}$$

This difference takes positive values for  $N \geq 2$ . This means that a country that cooperates with another country has no incentives to exit from the coalition. To check if a bilateral coalition is stable we also have to prove whether a third country has incentives to join the coalition and increase its size to three. Then, for  $n = 3$  we have

$$\pi_j^s(2) - \pi_i^s(3) = \frac{N^2 a^2 [8b^3 c^3 - Fb^2 c^4 - Gbc^5 - Hc^6]}{2b [(b + (N-2)c)^2 + 4bc]^2 [(b + (N-3)c)^2 + 9bc]} \quad (17)$$

$$\begin{aligned} F &\equiv (N^2 + 10N - 23), \quad G \equiv 2(N-2)(N-1)^2, \\ H &\equiv (N-3)^2(N-2)^2, \end{aligned}$$

where the term into square brackets in the numerator can be written as a function of  $\gamma \equiv b/c : c^6 [8\gamma^3 - F\gamma^2 - G\gamma - H]$ . It is easy to prove that this function decreases first until reaching a minimum and then it becomes increasing and convex. It is also easy to show that for  $\gamma = N(N-2)$  the value of this function is positive, so that, if the condition  $b/c > N(N-2)$  is satisfied, the function of  $\gamma$  into square brackets in the numerator of (17) is positive and, consequently,  $\pi_j^s(2) > \pi_i^s(3)$ . Therefore, an IEA consisting of two countries satisfies the stability condition established in Definition 1.

In Fig. 2, built from the numerical example of the previous section, we can see that starting from the great coalition, each country has incentives to exit from the agreement and this happens for any size of the coalition larger than two. Then we can conclude the following:

**Proposition 3** *A Stackelberg-IEA consisting of two countries is the unique self-enforcing IEA, independently of the scope of the gains to full-cooperation and the number of countries.*

Again we find that a bilateral agreement causes a small reduction in aggregate emissions of 1.6% and a small increase in aggregate net benefits of 13.7% taking into account that full-cooperation represents an increase of 204.57% in net benefits. The countries belonging to the coalition are slightly better off than in the Cournot equilibrium since their net benefits are higher in a 0.1%.

Finally, we compare the two stable agreements obtained in this section. For emissions of signatories we have that

$$q_i^s - q_i^c = \frac{2N(N-2)[b + (N-2)c]c^2a}{b[(b + (N-2)c)^2 + 4bc][b + (N+2)c]} > 0, \text{ for } N > 2,$$

and, for nonsignatory countries

$$q_j^s - q_j^c = -\frac{4N(N-2)c^3a}{b[(b + (N-2)c)^2 + 4bc][b + (N+2)c]} < 0 \text{ for } N > 2.$$

If we compare aggregate emissions

$$Q^s - Q^c = \frac{4N(N-2)c^2a^2}{[(b + (N-2)c)^2 + 4bc][b + (N+2)c]} > 0 \text{ for } N > 2.$$

Thus, we can conclude the following:

**Proposition 4** *Emissions by signatories are greater under leadership than under Cournot conjecture. Emissions by nonsignatories are smaller when they are followers. Aggregate emissions are greater in the Stackelberg equilibrium than in the Cournot equilibrium.*

It is clear from the above results that the net benefits of nonsignatory countries are greater in the Cournot equilibrium because these countries emit more whereas aggregate emissions and, consequently costs, are smaller than in the Stackelberg equilibrium. Regarding signatory countries we have that

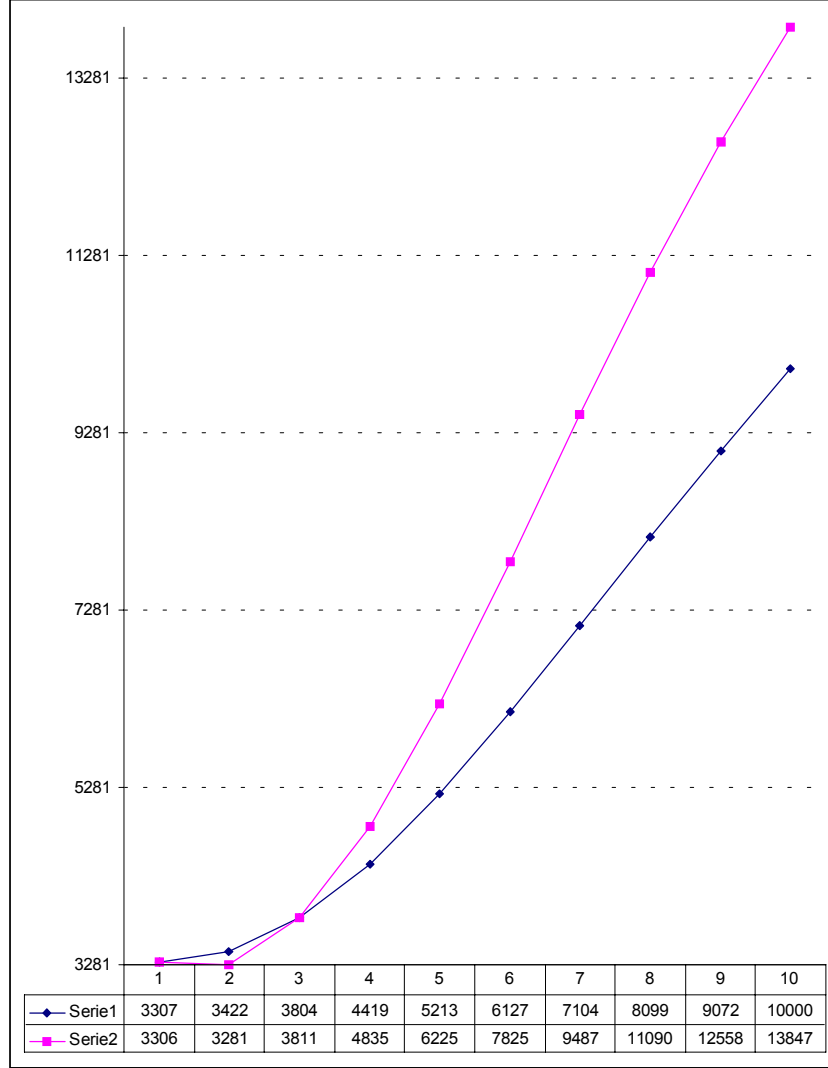
$$\pi_i^s - \pi_i^c = \frac{4N^2 (N-2)^2 c^4 a^2}{2b [(b + (N-2)c)^2 + 4bc] [b + (N+2)c]^2} > 0, \text{ for } N > 2$$

and, then, we can state:

**Proposition 5** *The net benefits of signatories are greater in the Stackelberg equilibrium than in the Cournot equilibrium.*

Although signatory countries face greater costs in the Stackelberg equilibrium than in the Cournot equilibrium, they obtain greater benefits because they emit more so that the total effect is an increase in net benefits. Thus, leadership favors to the countries that cooperate but generates a higher level of aggregate emissions and, consequently, a smaller level of environmental quality.

**Figure 2: Stable coalition under leadership.**



$N=10$ ,  $a=1000$ ,  $b=25$  and  $c=0.25$

Serie 1:  $\pi_i(n) \equiv$  net benefits of signatory countries for a coalition of size  $n$ .

Serie 2:  $\pi_j(n-1) \equiv$  net benefits of nonsignatory countries for a coalition of size  $n-1$ .

## 4 THE DYNAMIC MODEL

The main difference regarding the static model is that now the damage function depends on accumulated emissions,  $z$ ,

$$C_i(z) = \frac{c}{2}z^2. \quad (18)$$

Then, net benefits for each country are

$$\pi_i(q_i, z) = aq_i - \frac{b}{2}q_i^2 - \frac{c}{2}z^2.$$

Moreover, for a positive rate of natural decay  $k$ , the dynamic of accumulated emissions is given by the following differential equation

$$\dot{z} = \sum_{i=1}^N q_i - kz. \quad (19)$$

### 4.1 Full-cooperation

The cooperative outcome arises if all countries choose a level of emissions such that aggregate net benefits are maximized. Then, the coalition faces the following optimal control problem assuming the same emissions for all countries

$$\begin{aligned} \max_{\{q_i\}} \int_0^\infty e^{-\delta t} N \left( aq_i - \frac{b}{2}q_i^2 - \frac{c}{2}z^2 \right) dt \\ s.t. \quad \dot{z} = Nq_i - kz, \quad z(0) = z_0 \geq 0, \end{aligned} \quad (20)$$

where  $\delta$  is the discount rate. We implicitly assume the non-negativity constraint on the control variables and we do not impose  $z \geq 0$  as a state constraint but as a terminal condition,  $\lim_{t \rightarrow \infty} z \geq 0$ , for simplicity.

Defining the current value Hamiltonian in the standard way,

$$H(z, q_i, \lambda) = N \left( aq_i - \frac{b}{2}q_i^2 - \frac{c}{2}z^2 \right) + \lambda (Nq_i - kz),$$

the necessary conditions for an interior solution are

$$a - bq_i + \lambda = 0, \quad (21)$$

$$\dot{\lambda} = (\delta + k) \lambda + Ncz, \quad (22)$$

the transversality conditions being

$$\lim_{t \rightarrow \infty} e^{-\delta t} \lambda \geq 0, \lim_{t \rightarrow \infty} e^{-\delta t} \lambda z = 0. \quad (23)$$

The condition (21) establishes that the marginal benefit of emissions must be equal to the marginal user cost,  $\lambda$ . Using this condition to eliminate the marginal user cost from the Euler equation, we obtain the following differential equation for emissions

$$\dot{q}_i = (\delta + k) q_i + \frac{Nc}{b} z - \frac{(\delta + k) a}{b}$$

that together with the dynamic constraint of accumulated emissions allows us to obtain the optimal path of the variables of the problem if the transversality conditions hold. Among all the particular solutions of the system of differential equations  $\dot{z}$  and  $\dot{q}$ , we have the steady-state levels of the control and state variables

$$q_{i\infty}^* = \frac{k(\delta + k)a}{k(\delta + k)b + N^2c}, z_{\infty}^* = \frac{N(\delta + k)a}{k(\delta + k)b + N^2c} . \quad (24)$$

Now, in order to determine the stability features of the steady state we examine the characteristic roots of the system and obtain that the determinant of the Jacobian matrix is negative

$$\begin{vmatrix} \delta + k & \frac{Nc}{b} \\ N & -k \end{vmatrix} = - \left[ (\delta + k)k + \frac{N^2c}{b} \right] < 0.$$

This implies that the two roots have opposite signs, which establishes that the steady state of the system is a saddle point. For these types of critical points there are two stable branches in the phase diagram and there exists an optimal path to reach the steady state as illustrated in Fig. 3.

The optimal path can be calculated through standard methods:<sup>12</sup>

$$q_i^* = q_{i\infty}^* - \frac{Nc}{(r^* - \delta - k)b} z_\infty^* e^{r^* t}, \quad z^* = z_\infty^* (1 - e^{r^* t}), \quad (25)$$

where  $r^* = \frac{1}{2} \left[ \delta - \sqrt{\delta^2 + \frac{4}{b} [k(\delta + k)b + N^2 c]} \right] < 0$ .

All this can be summarized in the following proposition:

**Proposition 6** *For the optimal control problem (20) : i) There exists a unique steady state given by (24) and the optimal path defined by (25) leads to the steady state. ii) The steady state is a saddle point equilibrium and the optimal path approaches it asymptotically. iii) Initial emissions are higher than the steady-state emissions and emissions are decreasing along the optimal path.*

Finally, we can calculate the discounted present value of the flow of net benefits using (25).<sup>13</sup>

$$\begin{aligned} W_i^* &= \int_0^\infty e^{-\delta t} \left[ a q_i^* - \frac{b}{2} (q_i^*)^2 - \frac{c}{2} (z^*)^2 \right] dt = \\ &= \frac{\pi_{i\infty}^*}{\delta} + \frac{c}{b(r^* - \delta - k)} K (z_\infty^*)^2. \end{aligned} \quad (26)$$

It is easy to prove that  $K$  is negative which implies that the second term on the right-hand side in (26) is positive so that if the steady-state net benefits are positive, then,  $W_i^*$  is also positive. However, this condition is not necessary for getting a positive present value.

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<sup>12</sup>We assume without loss of generality that  $z_0 = 0$ .

<sup>13</sup>See Appendix B.

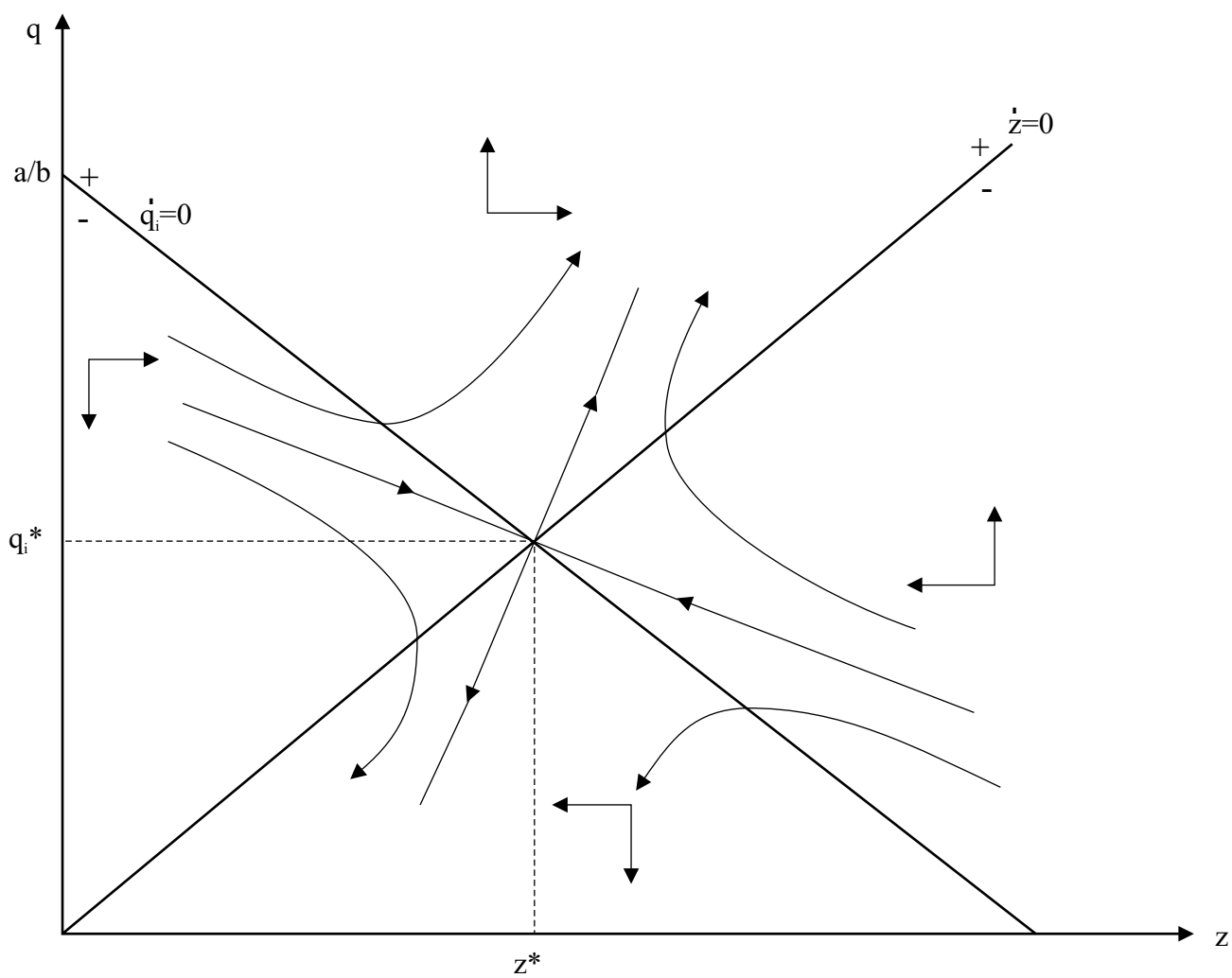


Figure 3: Phase diagram of the cooperative solution.



## 4.2 Open-loop Nash equilibrium

When countries do not cooperate they choose the level of emissions that maximizes their own net benefits. Then, each country faces the following optimal control problem

$$\begin{aligned} \max_{\{q_i\}} \int_0^\infty e^{-\delta t} \left( a q_i - \frac{b}{2} q_i^2 - \frac{c}{2} z^2 \right) dt \\ s.t. \quad \dot{z} = \sum_{j=1}^N q_j - k z, \quad z(0) = z_0 \geq 0, \end{aligned} \quad (27)$$

so that the noncooperative equilibrium is the solution of the differential game defined by (27) for  $i = 1, \dots, N$ .

In this section we compute the open-loop Nash equilibrium of the game. For the open-loop Nash equilibrium, countries commit themselves at the moment of starting to an entire temporal path of emissions that maximizes the present value of their stream of net benefits given the emission path of the rival countries. Then for every given path  $q_j$  of country  $j$ ,  $j = 1, \dots, N-1$  country  $i$  faces the problem of maximizing (27). The other countries  $j$  face a similar problem. An equilibrium of the game are  $N$  open-loop strategies that solve the  $N$  optimization problems simultaneously. Defining the current value Hamiltonian in the standard way,

$$H(z, q_1, q_2, \dots, q_N, \lambda_i) = a q_i - \frac{b}{2} q_i^2 - \frac{c}{2} z^2 + \lambda_i \left( \sum_{j=1}^N q_j - k z \right), i = 1, \dots, N.$$

The necessary conditions for an interior open-loop equilibrium are

$$a - b q_i + \lambda_i = 0, i = 1, \dots, N, \quad (28)$$

$$\dot{\lambda}_i = (\delta + k) \lambda_i + c z, i = 1, \dots, N, \quad (29)$$

the transversality conditions being

$$\lim_{t \rightarrow \infty} e^{-\delta t} \lambda_i \geq 0, \lim_{t \rightarrow \infty} e^{-\delta t} \lambda_i z = 0. \quad (30)$$

The condition (28) establishes that the marginal benefit of emissions must be equal to the *private* marginal user cost,  $\lambda_i$ . This cost evolves according to the Euler equation (29). If we compare the Euler equations (22) and (29), we realize that the difference between them is given by  $(N - 1) cz$  which represents the *social cost* of a marginal increment in accumulated emissions. In the noncooperative equilibrium, countries do not take this cost into account. Note that this cost varies directly with the number of countries.

Assuming symmetric countries simplifies the solution. With symmetry,  $q_i = q_j$  and  $\lambda_i = \lambda_j$  and, therefore, the  $2N$  equations defined by (28) and (29) reduces to 2. Then, using the condition (28) to eliminate the marginal user cost from (29), we obtain the following differential equation for emissions

$$\dot{q}_i = (\delta + k) q_i + \frac{c}{b} z - \frac{(\delta + k) a}{b}$$

that together with the dynamic constraint of accumulated emissions allows us to calculate the optimal path of the open-loop Nash equilibrium. The particular solution  $\dot{z} = \dot{q} = 0$  yields the steady-state values of the control and state variables<sup>14</sup>

$$q_{i\infty}^{ol} = \frac{k(\delta + k)a}{k(\delta + k)b + Nc}, \quad z_{\infty}^{ol} = \frac{N(\delta + k)a}{k(\delta + k)b + Nc}. \quad (31)$$

Furthermore, the determinant of the Jacobian matrix of the system of differential equations is negative

$$\begin{vmatrix} \delta + k & \frac{c}{b} \\ N & -k \end{vmatrix} = - \left[ (\delta + k)k + \frac{Nc}{b} \right] < 0.$$

This implies that the steady state of the system is a saddle point and that there exists an optimal path which leads to the steady state. The phase diagram is similar to the one of the cooperative solution, see Fig. 3. However,

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<sup>14</sup>Where *ol* stands for open-loop Nash equilibrium.

for the noncooperative solution the  $\dot{q}_i = 0$  locus is above the  $\dot{q}_i = 0$  locus corresponding to the cooperative solution except for  $z = 0$  resulting in a steady-state equilibrium where both emissions and accumulated emissions are higher than in the cooperative equilibrium.

The optimal path can be calculated through standard methods:

$$q_i^{ol} = q_{i\infty}^{ol} - \frac{c}{(r^{ol} - \delta - k)b} z_{\infty}^{ol} e^{r^{ol}t}, \quad z^{ol} = z_{\infty}^{ol} (1 - e^{r^{ol}t}), \quad (32)$$

where  $r^{ol} = \frac{1}{2} \left[ \delta - \sqrt{\delta^2 + \frac{4}{b} [k(\delta + k)b + Nc]} \right] < 0$ .

All this can be summarized in the following proposition:

**Proposition 7** *For the differential game defined by (27) for  $i = 1, \dots, N$  :*  
*i) There exists a unique steady state given by (31) and the optimal path defined by (32) leads to the steady state. ii) The steady state is a saddle point equilibrium and the optimal path approaches it asymptotically. iii) Initial emissions are higher than steady-state emissions and emissions are decreasing along the optimal path.*

Finally, we can calculate the discounted present value of the flow of net benefits substituting (32) into (27):

$$\begin{aligned} W_i^{ol} &= \int_0^{\infty} e^{-\delta t} \left[ a q_i^{ol} - \frac{b}{2} (q_i^{ol})^2 - \frac{c}{2} (z^{ol})^2 \right] dt = \\ &= \frac{\pi_{i\infty}^{ol}}{\delta} + \frac{c}{b(r^{ol} - \delta - k)} L (z_{\infty}^{ol})^2, \end{aligned} \quad (33)$$

where

$$L = \frac{c - b(r^{ol} - \delta - k)(\delta + k)}{(r^{ol} - \delta)(\delta + k)} + \frac{c + b(r^{ol} - \delta - k)^2}{2(2r^{ol} - \delta)(r^{ol} - \delta - k)}.$$

The second term on the right-hand side of (33) is positive as happened in the cooperative solution.

To end this subsection we compare the two solutions. We have already established that the cooperative solution is characterized by a lower steady-state stock of pollution and a lower level of emissions. Next, we compare the

optimal paths of emissions and accumulated emissions. From (25) and (32) we obtain that

$$\dot{z}^* = \frac{dz^*}{dt} = z_\infty^* |r^*| e^{r^* t} \text{ and } \dot{z}^{ol} = \frac{dz^{ol}}{dt} = z_\infty^{ol} |r^{ol}| e^{r^{ol} t}$$

where  $|r^*| < |r^{ol}|$ . Let's assume that  $\dot{z}^{ol} \leq \dot{z}^*$ , then  $z_\infty^{ol} |r^{ol}| e^{r^{ol} t} \leq z_\infty^* |r^*| e^{r^* t}$  that can be rewritten as

$$z_\infty^{ol} |r^{ol}| \leq z_\infty^* |r^*| e^{(r^* - r^{ol})t} \quad (34)$$

where  $r^* - r^{ol} < 0$ . So that at the steady state, i.e., when  $t \rightarrow \infty$ , (34) implies that  $z_\infty^{ol} |r^{ol}| \leq 0$  yielding a contradiction. Thus, we can conclude that  $\dot{z}^*(t) < \dot{z}^{ol}(t)$  and as  $z^*(0) = z^{ol}(0) = z_0 = 0$  we obtain that  $z^*(t) < z^{ol}(t)$  for all  $t \in (0, \infty)$ . In order to compare the time paths of emissions we use the dynamic constraint of accumulated emissions. The previous result implies that

$$\dot{z}^{ol} = Nq_i^{ol} - kz^{ol} > \dot{z}^* = Nq_i^* - kz^*,$$

for all  $t$ . Rearranging terms, we can write

$$N(q_i^{ol} - q_i^*) > k(z^{ol} - z^*) \geq 0$$

since  $z^{ol}(t) \geq z^*(t)$ , for all  $t \in [0, \infty)$ , which implies that  $q_i^{ol}(t) > q_i^*(t)$  for all  $t \in [0, \infty)$ . Consequently, we can write the following proposition:

**Proposition 8** *Both emissions and accumulated emissions for the open-loop Nash equilibrium are higher than for the cooperative equilibrium for all  $t \in (0, \infty)$ . Moreover, initial emissions for the noncooperative solution are higher than for the cooperative solution.*

Finally, we would want to point out that the difference between the cooperative and the noncooperative outcome decreases with respect to  $b/c$ . If we

calculate the percentage variation of accumulated emissions we obtain the following expression

$$\frac{z_{\infty}^* - z_{\infty}^{ol}}{z_{\infty}^{ol}} = -\frac{N(N-1)}{(b/c)k(\delta+k) + N^2}, \quad (35)$$

whose absolute value negatively depends on  $b/c$ . This result establishes that an increase in the environmental damages results in an increase in the inefficiency of the noncooperative solution, as was to be expected.

### 4.3 Feedback Nash equilibrium

A feedback strategy consists of a contingency plan that indicates the optimal value of the control variable for each value of the state variable at each point in time. Thus, feedback strategies have the property of being *subgame perfect* because after each player's actions have caused the state of the system to evolve from its initial state to a new state, the continuation of the game may be regarded as a subgame of the original game. Therefore, a feedback strategy must satisfy the principle of optimality of dynamic programming

$$\delta W_i = \max_{\{q_i\}} \left\{ a q_i - \frac{b}{2} q_i^2 - \frac{c}{2} z^2 + W_i' \left[ \sum_{j=1}^N q_j - k z \right] \right\}, \quad (36)$$

where  $W_i(z)$  stands for the optimal control value function associated with the optimization problem (27), i.e. it denotes the maximum discounted present value of the flow of net benefits subject to the dynamic constraint of accumulated emissions for the current value of the state variable, and  $W_i'$  is its first derivative.

>From the first order condition for the maximization of the right-hand side of the Bellman equation, we get

$$a - b q_i + W_i' = 0, \quad i = 1, \dots, N \quad (37)$$

This condition is equivalent to the one obtained for the open-loop Nash equilibrium and can be given the same interpretation. See (28). However, now this condition defines the optimal strategy for emissions as a function of accumulated emissions:  $q_i = (a + W_i'(z)) / b$ . Next, by incorporating this

optimal strategy into the Bellman equation, we eliminate the maximization and obtain, after a number of calculations, a nonlinear differential equation

$$\delta W_i = \frac{1}{2b} (a + W'_i)^2 + \frac{N-1}{b} W'_i (a + W'_i) - W'_i k z - \frac{c}{2} z^2. \quad (38)$$

In order to derive the solution to this equation, we guess a quadratic representation for the value function  $W_i$

$$W_i(z) = \frac{1}{2} \alpha_i z^2 + \beta_i z + \mu_i.$$

Substituting into (38) and equating coefficients yields the following set of equations

$$(2N-1) \alpha_i^2 - 2b \left( k + \frac{\delta}{2} \right) \alpha_i - bc = 0 \quad (39)$$

$$Na\alpha_i + [(2N-1) \alpha_i - b(k + \delta)] \beta_i = 0 \quad (40)$$

$$(a + \beta_i) [a + (2N-1) \beta_i] - 2b\delta\mu_i = 0 \quad (41)$$

The solution to this system is given by the following values

$$\alpha_i = \frac{b}{2N-1} \left[ k + \frac{\delta}{2} \pm \sqrt{\left( k + \frac{\delta}{2} \right)^2 + \frac{2N-1}{b} c} \right], \quad (42)$$

$$\beta_i = \frac{Na\alpha_i}{b(k + \delta) - (2N-1) \alpha_i}, \quad (43)$$

$$\mu_i = \frac{a + \beta_i}{2b\delta} [a + (2N-1) \beta_i]. \quad (44)$$

This solution allows us to derive the linear feedback Nash equilibrium strategy for emissions

$$q_i = \frac{1}{b} (a + \beta_i + \alpha_i z), \quad i = 1, \dots, N \quad (45)$$

and to write the dynamic constraint as

$$\dot{z} = \frac{N}{b} (a + \beta_i) - \left( k - \frac{N}{b} \alpha_i \right) z \quad (46)$$

which can be used to calculate the optimal path of the feedback Nash equilibrium. The particular solution  $\dot{z} = 0$  yields the steady state for the state variable and using (45) we obtain the steady-state value for the control variable:<sup>15</sup>

$$z_\infty^f = \frac{N [b (k + \delta) - (N - 1) \alpha_i] a}{b [k (b (k + \delta) - (N - 1) \alpha_i) + Nc]}, \quad (47)$$

$$q_{i\infty}^f = \frac{k [b (k + \delta) - (N - 1) \alpha_i] a}{b [k (b (k + \delta) - (N - 1) \alpha_i) + Nc]}. \quad (48)$$

Next, using the stability condition for (46) we select one of the two solutions given by (42). For the differential equation (46), the stability condition is

$$\frac{d\dot{z}}{dz} = - \left( k - \frac{N}{b} \alpha_i \right) < 0.$$

The negative root of (42) satisfies this condition but not the positive root. Using this negative root we can check that  $a + \beta_i$  in (45) is positive

$$a + \beta_i = \frac{[b (k + \delta) - (N - 1) \alpha_i] a}{b (k + \delta) - (2N - 1) \alpha_i},$$

and can conclude that the linear strategy (45) defines a negative relationship between current and accumulated emissions.

Now, solving the differential equation (46) and using (45) we obtain the optimal path for emissions and accumulated emissions

$$q_i^f = q_{i\infty}^f - \frac{\alpha_i}{b} z_\infty^f e^{-(k - \frac{N}{b} \alpha_i)t}, \quad z^f = z_\infty^f \left[ 1 - e^{-(k - \frac{N}{b} \alpha_i)t} \right], \quad (49)$$

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<sup>15</sup>Where  $f$  stands for the feedback Nash equilibrium. We have used (39) in order to simplify the final expression.

where  $\alpha_i = \frac{b}{2N-1} \left[ k + \frac{\delta}{2} - \sqrt{\left(k + \frac{\delta}{2}\right)^2 + \frac{2N-1}{b}c} \right] < 0$ .

All this can be summarized in the following proposition:

**Proposition 9** *For the differential game defined by (27) for  $i = 1, \dots, N$ :*  
*i) There exists a unique linear equilibrium strategy given by (45). ii) The steady state defined by (47) and (48) is unique and the optimal path given by (49) approaches it asymptotically. iii) The steady state is globally stable. iv) Initial emissions are higher than steady-state emissions and emissions are decreasing along the optimal path.*

For the feedback Nash equilibrium, the discounted present value of the net benefits stream, if we assume that  $z_0 = 0$ , is directly given by  $\mu_i$  that can be written in terms of  $\alpha_i$

$$W_i^f = \mu_i = \frac{[b(k + \delta) - (N - 1)\alpha_i][b(k + \delta) + (2N - 1)(N - 1)\alpha_i]a^2}{2\delta b[b(k + \delta) - (2N - 1)\alpha_i]^2}.$$

To end this section we compare the two noncooperative solutions. We prove in Appendix C the following proposition:

**Proposition 10** *Both emissions and accumulated emissions for the feedback Nash equilibrium in linear strategies are higher than for the open-loop Nash equilibrium for all  $t \in (0, \infty)$ . Moreover, initial emissions for the feedback Nash equilibrium are higher than for the open-loop Nash equilibrium.*

This result establishes that the noncooperative outcome is less efficient for the feedback Nash equilibrium than for the open-loop Nash equilibrium. This bias of the feedback Nash equilibrium in linear strategies also appears in the papers written by van der Ploeg and de Zeeuw (1992) and Mäler and de Zeeuw (1998) and has been obtained in other applications of differential games, see, for instance, Rubio and Casino (2001) and the literature there quoted. The bias appears because when agents play feedback strategies they can immediately react to any deviation or change in emissions of the other agents as long as emissions depend on accumulated emissions and the dynamics of this variable is determined by emissions of all agents. With feedback strategies the strategic interdependence among the agents is stressed and the



efficiency losses of the noncooperative outcome increase with respect to the open-loop Nash equilibrium. The intuition, using de Zeeuw's words, is as follows: "Each country knows that in a feedback information structure the other countries observe the stock of pollutants and react to higher stocks with lower output and pollution. Therefore, each country knows that an increase in output and pollution will then be partly offset by a decrease in all the other countries. This implies that the feedback equilibrium will lead to higher levels of pollution than the open-loop equilibrium, where the countries do not observe the stock of pollutants" (de Zeeuw (1998, p. 251)).

Finally, we would want to point out that the difference between the cooperative and the noncooperative outcome decreases with respect to  $b/c$ . If we calculate the percentage variation of accumulated emissions we obtain the following expression

$$\frac{z_{\infty}^* - z_{\infty}^f}{z_{\infty}^f} = -\frac{N(N-1)[\delta + k - N(\alpha_i/b)]}{(k(\delta + k)(b/c) + N^2)(\delta + k - (N-1)(\alpha_i/b))}. \quad (50)$$

This expression depends on  $b/c$  since

$$\frac{\alpha_i}{b} = \frac{1}{2N-1} \left[ k + \frac{\delta}{2} - \sqrt{\left(k + \frac{\delta}{2}\right)^2 + \frac{2N-1}{(b/c)}} \right].$$

It is easy to show that the derivative of (50) with respect to  $b/c$  is positive, see Appendix D. This result establishes that an increase of the environmental damages results in an increase of the inefficiency of the noncooperative solution.

The percentage variation of steady-state accumulated emissions for the different noncooperative solutions

$$\frac{z_{\infty}^{ol} - z_{\infty}^f}{z_{\infty}^f} = \frac{N(N-1)(\alpha_i/b)}{(k(\delta + k)(b/c) + N)(\delta + k - (N-1)(\alpha_i/b))} \quad (51)$$

also decreases in absolute values with respect to  $b/c$ . This means that the open-loop Nash equilibrium is not an accurate approximation of the feedback Nash equilibrium if the environmental damages are high enough.

## 5 STABILITY OF AN IEA IN THE DYNAMIC MODEL

The stability of an IEA in a dynamic model can be analyzed using a differential game among  $n$  signatory countries and  $N - n$  nonsignatory countries. In this section we calculate the open-loop Nash equilibrium and the feedback Nash equilibrium of the game and we look for the stable coalition in a dynamic framework.

### 5.1 Open-loop Nash equilibrium

We assume that each nonsignatory country chooses the level of emissions that maximizes the present value of the stream of net benefits given the emissions path of the rival countries including the signatories

$$\max_{\{q_j\}} \int_0^\infty e^{-\delta t} \left( a q_j - \frac{b}{2} q_j^2 - \frac{c}{2} z^2 \right) dt. \quad (52)$$

Signatory countries also take the emissions of nonsignatories as given and commit to a level of emissions such that

$$\max_{\{q_i\}} \int_0^\infty e^{-\delta t} n \left( a q_i - \frac{b}{2} q_i^2 - \frac{c}{2} z^2 \right) dt. \quad (53)$$

In both cases, countries face the same dynamic constraint

$$\dot{z} = \sum_{i=1}^n q_i + \sum_{j=1}^{N-n} q_j - k z, \quad z(0) = z_0 \geq 0. \quad (54)$$

As in the previous section, an equilibrium of the game are  $N$  open-loop strategies that solve the  $N$  optimization problems simultaneously. Defining the current value of the Hamiltonian in the standard way, we obtain the following set of necessary conditions for an interior open-loop equilibrium

$$a - b q_i + \lambda_i = 0, \quad i = 1, \dots, n, \quad (55)$$

$$\dot{\lambda}_i = (\delta + k) \lambda_i + nc z, \quad i = 1, \dots, n, \quad (56)$$

$$a - bq_j + \lambda_j = 0, \quad j = 1, \dots, N - n, \quad (57)$$

$$\dot{\lambda}_j = (\delta + k) \lambda_j + cz, \quad j = 1, \dots, N - n \quad (58)$$

the transversality conditions being

$$\lim_{t \rightarrow \infty} e^{-\delta t} \lambda_i \geq 0, \quad \lim_{t \rightarrow \infty} e^{-\delta t} \lambda_i z = 0, \quad (59)$$

$$\lim_{t \rightarrow \infty} e^{-\delta t} \lambda_j \geq 0, \quad \lim_{t \rightarrow \infty} e^{-\delta t} \lambda_j z = 0. \quad (60)$$

Each kind of country faces a different marginal user cost. For nonsignatories,  $\lambda_j$  is the *private* marginal user cost, whereas for signatories,  $\lambda_i$  represents the marginal user cost for the countries that participate in the IEA ( $n \leq N$ ). Obviously, for  $n = N$ , (55) and (56) are identical to the necessary conditions of the cooperative solution, and for  $n = 0$ , (57) and (58) are identical to the necessary conditions of the noncooperative solution.

Under the symmetry assumption the  $2N$  equations defined by (55) – (58) reduce to 4. Then, using (55) and (57) to eliminate the marginal user cost from (56) and (58), we obtain the following system of differential equations for emissions

$$\dot{q}_i = (\delta + k) q_i + \frac{nc}{b} z - \frac{(\delta + k) a}{b},$$

$$\dot{q}_j = (\delta + k) q_j + \frac{c}{b} z - \frac{(\delta + k) a}{b},$$

that together with the dynamic constraint of accumulated emissions allows us to calculate the optimal path of the open-loop Nash equilibrium. The

particular solution  $\dot{q}_i = \dot{q}_j = \dot{z} = 0$  yields the steady-state values of the control and state variables<sup>16</sup>

$$q_{i\infty}^{ol} = \frac{a[k(\delta+k)b - (N-n)(n-1)c]}{b[k(\delta+k)b + (N+n^2-n)c]}, \quad (61)$$

$$q_{j\infty}^{ol} = \frac{a[k(\delta+k)b + n(n-1)c]}{b[k(\delta+k)b + (N+n^2-n)c]}, \quad (62)$$

$$z_{\infty}^{ol} = \frac{N(\delta+k)a}{k(\delta+k)b + (N+n^2-n)c}. \quad (63)$$

It is easy to see from (63) that the steady-state accumulated emissions decrease as the number of countries that sign the agreement increases. It is also easy to prove that the steady state emissions of nonsignatory countries increase with the number of signatories:

$$\frac{\partial q_{j\infty}^{ol}}{\partial n} = \frac{Nac^2(2n-1)}{b[k(\delta+k)b + (N+n^2-n)c]^2} > 0.$$

However, the steady-state emissions of signatory countries can increase or decrease, although the aggregate state emissions always decrease with respect to the number of countries belonging to the agreement.

Furthermore, the determinant of the Jacobian matrix of the system of differential equations is negative

$$\begin{vmatrix} \delta+k & 0 & \frac{nc}{b} \\ 0 & \delta+k & \frac{c}{b} \\ n & N-n & -k \end{vmatrix} = -(\delta+k) \left[ n(\delta+k) + \frac{(N+n^2-n)c}{b} \right] < 0.$$

---

<sup>16</sup>A necessary and sufficient condition for positive steady-state emissions of signatories for all  $n$  in the interval  $[2, N-1]$  is  $(b/c) > (N-1)^2/4k(\delta+k)$ . From (61) we obtain that  $q_{i\infty}^{ol} > 0$  implies that  $(b/c) > (N-n)(n-1)/k(\delta+k)$ , where  $(N-n)(n-1)$  is a concave function with respect to  $n$  which reaches a maximum for  $n = (N+1)/2$  equal to  $(N-1)^2/4$ . By substitution of this maximum in the previous inequality we get the lower bound for  $b/c$  which guarantees that steady-state emissions of signatories are positive for all  $n$ .

This implies that the steady-state of the system is a saddle point and that there exists an optimal path which leads to it. The optimal paths for the different variables can be calculated through standard methods:

$$q_i^{ol} = q_{i\infty}^{ol} - \frac{nc}{b(r - \delta - k)} z_{\infty}^{ol} e^{rt}, \quad (64)$$

$$q_j^{ol} = q_{j\infty}^{ol} - \frac{c}{b(r - \delta - k)} z_{\infty}^{ol} e^{rt}, \quad (65)$$

$$z^{ol} = z_{\infty}^{ol} (1 - e^{rt}), \quad (66)$$

where  $r = \frac{1}{2} \left[ \delta - \sqrt{\delta^2 + \frac{4}{b} [k(\delta + k)b + (N + n^2 - n)c]} \right] < 0$ .

Thus, the open-loop Nash equilibrium of the game has the same features that the solution for the differential game studied in the previous section. For this reason, we omit to summarize them (see Proposition 7).

Next, we show that:

**Proposition 11** *The discounted present value of nonsignatories is higher than the discounted present value of signatories:  $W_j^{ol}(n) > W_i^{ol}(n)$ .*

**Proof.** First, we compare the temporal paths of emissions given by (64) and (65) beginning with the initial values

$$q_i^{ol}(0) - q_j^{ol}(0) = -(n-1) \left[ \frac{Nca}{b[(N + n^2 - n)c + k(\delta + k)b]} + \frac{c}{b(r - \delta - k)} z_{\infty}^{ol} \right].$$

The first term between square brackets can be written, using (63), as follows

$$\frac{Nca}{b[(N + n^2 - n)c + k(\delta + k)b]} = \frac{c}{b(\delta + k)} z_{\infty}^{ol},$$

which yields

$$q_i^{ol}(0) - q_j^{ol}(0) = -\frac{(n-1)cr}{b(\delta + k)(r - \delta - k)} z_{\infty}^{ol} < 0.$$

Then, as  $dq_i^{ol}/dt = n(dq_j^{ol}/dt) < 0$ , we can conclude that the emissions of nonsignatory countries are higher than the emissions of signatories for all  $t \in [0, \infty)$ , which implies, as can be easily checked from (61) and (62), that  $q_{i\infty}^{ol} < q_{j\infty}^{ol}$ . Another implication of this conclusion is that nonsignatories obtain a higher payoff than signatories. Notice that both types of countries face the same costs as long as  $z$  is a public bad and there is symmetry in the benefit and cost functions.  $\square$

However, in order to study the stability of the coalition we need to compare  $W_i^{ol}(n)$  and  $W_j^{ol}(n-1)$ . Substituting the optimal control paths of emissions and pollution stock, given by (64), (65) and (66) into (52) and (53) and integrating, we obtain the discounted present value of net benefits for the signatories and nonsignatories

$$W_i^{ol} = \frac{\pi_{i\infty}^{ol}}{\delta} + \frac{c}{b(r-\delta-k)} P(z_\infty^{ol})^2,$$

where

$$P = \frac{cn^2 - b(r-\delta-k)(\delta+k)}{(r-\delta)(\delta+k)} + \frac{cn^2 + b(r-\delta-k)^2}{2(2r-\delta)(r-\delta-k)}.$$

and

$$W_j^{ol} = \frac{\pi_{j\infty}^{ol}}{\delta} + \frac{c}{b(r-\delta-k)} Q(z_\infty^{ol})^2,$$

where

$$Q = \frac{c - b(r-\delta-k)(\delta+k)}{(r-\delta)(\delta+k)} + \frac{c + b(r-\delta-k)^2}{2(2r-\delta)(r-\delta-k)}.$$

## 5.2 A numerical example

Unfortunately, it is not possible to compare analytically  $W_i^{ol}(n)$  and  $W_j^{ol}(n-1)$ . However, a numerical example shows that the discounted value of net benefits increases with respect to  $n$  for both signatory and nonsignatory countries,

and that only a bilateral coalition can be stable as long as  $W_i^{ol}(2) > W_j^{ol}(1)$  and  $W_j^{ol}(n-1) > W_i^{ol}(n)$  for any size of coalition higher than two,<sup>17</sup> see Table 2.

**Table 2: Stability analysis for the numerical example. Open-loop strategies\***

n	$q_{i\infty}$	$q_{j\infty}$	$z_{\infty}$	$W_i$	$W_j$	$W$
1		26.087	52173.913		42494805	424948050
2	23.687	26.129	51282.051	42535812	42673667	426460960
3	21.488	26.210	49586.777	42662880	43020919	429135073
4	19.573	26.322	47244.094	42871247	43516977	432586850
5	17.989	26.455	44444.444	43153388	44135677	436445325
6	16.749	26.601	41379.310	43499700	44847225	440387100
7	15.833	26.752	38216.561	43899272	45621198	444158498
8	15.205	26.901	35087.719	44340702	46429025	447583666
9	14.821	27.044	32085.561	44812801	47245665	450560874
10	14.634		29268.293	45305098		453050980

\*Assumes  $k=0.005$ ,  $N=10$ ,  $a=100000$ ,  $b=3500$ ,  $c=0.005$  and  $\delta=0.025$

Finally, in order to address the *scope* of this result we develop a sensitivity analysis considering different values for the parameters of the model. Table 3 presents the values chosen.

<sup>17</sup>In order to avoid a negative discounted present value of the flow of net benefits we have had to increase substantially the ratio  $b/c$  with respect to the static model. We have not used these new values for the static model because in that case the numerical differences between the Cournot and Stackelberg equilibria are imperceptible.

**Table 3: Parameter values for the sensitivity analysis and reductions in steady-state accumulated emissions (%). Open-loop strategies.\***

b	c				
	0.0010	0.0025	0.0050	0.0075	0.0100
1500	27.69	47.37	62.07	69.23	73.47
	17.56	33.96	49.31	58.07	63.72
2500	18.95	36.00	51.43	60.00	65.46
	11.43	24.00	37.90	46.96	53.33
3500	14.40	29.03	43.90	52.94	59.02
	8.47	18.56	30.77	39.42	45.86
4500	11.61	24.32	38.30	47.37	53.73
	6.73	15.13	25.90	33.96	40.22
5500	9.73	20.93	33.96	42.86	49.31
	5.58	12.77	22.36	29.83	35.82

\* Assumes  $k=0.005$ ,  $N=10$  and  $a=100000$ . The top number in each cell represents the gains of cooperation for  $\delta = 0.025$  and the bottom for  $\delta = 0.05$

In the cells of Table 3 we write the percentage reductions in steady-state accumulated emissions given by (35) for two different discount rates. These variations gives us an idea of the potential gains coming from cooperation in terms of the reductions in the steady-state accumulated emissions that could be achieved in the case of a full-cooperation. The *maximum* gains of full cooperation are obtained when  $c = 0.01$ ,  $b = 1500$  and the rate of discount is equal to 0.025. This upper bound appears because we impose the following lower bound to the  $b/c$  ratio<sup>18</sup>:

$$\frac{b}{c} > \frac{(N-1)^2}{4k(\delta+k)}, \quad (67)$$

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<sup>18</sup>Notice that according to (35) the reductions in the steady-state accumulated emissions are inversely related with  $b/c$ .



which guarantees that  $q_{i\infty}^{ol}$  is positive for all  $n$ , see Footnote (16).<sup>19</sup> From this case we consider lower values for  $c$  and higher values for  $b$  until reaching a potential gains of around 10% for  $\delta = 0.025$  assuming that the issue of cooperation fails to be attractive below this value. This is the logic of Table 3 and the example. In this way, we think that the considered range of values for the parameters covers all the relevant situations for the study of the stability of the agreement. Our calculations yield the same result for the fifty cases studied: only a *bilateral* IEA is self-enforcing independently of the gains coming from cooperation. The similarity between the results obtained in the static model and the ones obtained in the dynamic model should not surprise us because when the open-loop Nash equilibrium concept is used to solve the differential game, this becomes essentially a one-shot game as long as the players commit themselves at the moment of starting to an entire temporal path of emissions. The nature of the game, therefore, does not change substantially and for this reason we ought not to expect qualitative differences in our conclusions.

### 5.3 Feedback Nash equilibrium

As in subsection 4.3 we assume that agents play linear strategies that satisfy the principle of optimality of dynamic programming. The Bellman's equation for nonsignatories is

$$\delta W_j = \max_{\{q_j\}} \left\{ a q_j - \frac{b}{2} q_j^2 - \frac{c}{2} z^2 + W_j' \left[ \sum_{i=1}^n q_i + \sum_{j=1}^{N-n} q_j - k z \right] \right\}, \quad (68)$$

and the optimal value of the control variable must satisfy the necessary condition

$$a - b q_j + W_j' = 0, j = 1, \dots, N - n. \quad (69)$$

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<sup>19</sup>In fact, the maximum reduction in steady-state accumulated emissions is something higher: 74.84% for  $\delta = 0.025$  if we assume that  $q_{i\infty}^{ol}$  is zero, that happens for  $b/c = 135000$  according to the values of  $N$  and  $k$  that appear in Table 2. Instead of this, we have assumed a strictly positive value for  $q_{i\infty}^{ol}$  with  $b/c = 150000$ . Then we have chosen an arbitrary value for  $c$  equal to 0.01 which yields  $b = 1500$ .

To calculate the linear strategies for signatories we use the following Bellman's equation

$$\delta W_n = \max_{\{q_i\}} \left\{ \sum_{i=1}^n \left[ a q_i - \frac{b}{2} q_i^2 - \frac{c}{2} z^2 \right] + W'_n \left[ \sum_{i=1}^n q_i + \sum_{j=1}^{N-n} q_j - k z \right] \right\} \quad (70)$$

where  $W_n$  stands for the value function of the coalition. From this equation we obtain the following necessary condition

$$a - b q_i + W'_n = 0, i = 1, \dots, n. \quad (71)$$

Substituting  $q_i = (a + W'_n) / b$  and  $q_j = (a + W'_j) / b$  in (70) and rearranging terms we find that

$$\delta W_n = \frac{n}{2b} (a + W'_n)^2 + \frac{N-n}{b} (a + W'_j) W'_n - k W'_n z - \frac{nc}{2} z^2. \quad (72)$$

Now, substituting the optimal values of  $q_i$  and  $q_j$  in (68) we get for  $j = 1, \dots, N - n$ ,

$$\begin{aligned} \delta W_j = & \frac{1}{2b} (a + W'_j)^2 + \frac{n}{b} (a + W'_n) W'_j - k W'_j z - \frac{c}{2} z^2 \\ & + \frac{N-n-1}{b} (a + W'_j) W'_j. \end{aligned} \quad (73)$$

In order to derive the solution to the differential equation system (72) and (73) we guess quadratic representations for the value functions  $W_n$  and  $W_j$ ,

$$W_n = \frac{1}{2} \alpha_n z^2 + \beta_n z + \mu_n, \quad W_j = \frac{1}{2} \alpha_j z^2 + \beta_j z + \mu_j. \quad (74)$$

Using (74) to eliminate  $W_n$ ,  $W'_n$ ,  $W_j$  and  $W'_j$  from Eqs. (72) and (73) and equating yields the following system of equations for the coefficients of the value functions.

$$n\alpha_n^2 + 2 \left[ (N-n)\alpha_j - b \left( k + \frac{\delta}{2} \right) \right] \alpha_n - nbc = 0 \quad (75)$$

$$Na\alpha_n + (N-n)\alpha_n\beta_j + [n\alpha_n + (N-n)\alpha_j - b(k+\delta)]\beta_n = 0 \quad (76)$$

$$n(a+\beta_n)^2 + 2(N-n)(a+\beta_j)\beta_n - 2b\delta\mu_n = 0 \quad (77)$$

$$[2(N-n)-1]\alpha_j^2 + 2 \left[ n\alpha_n - b \left( k + \frac{\delta}{2} \right) \right] \alpha_j - bc = 0 \quad (78)$$

$$Na\alpha_j + n\alpha_j\beta_n + [(2(N-n)-1)\alpha_j + n\alpha_n - b(k+\delta)]\beta_j = 0 \quad (79)$$

$$(a+\beta_j)[a+(2(N-n)-1)\beta_j] + 2n(a+\beta_n)\beta_j - 2b\delta\mu_j = 0 \quad (80)$$

This system does not have an analytical solution. For this reason, we use the same numerical example that the one used for the open-loop Nash equilibrium to calculate the solution.<sup>20</sup> Eqs.(75) and (78) have two pairs of real solutions for  $\alpha_n$  and  $\alpha_j$ . A pair of positive values and another pair of negative values but only the negative values satisfy the stability condition. To obtain this condition we substitute the linear strategies in the dynamic constraint of accumulated emissions which yields the following differential equation

$$\dot{z} = \frac{Na}{b} + \frac{n\beta_n + (N-n)\beta_j}{b} - \left( k - \frac{n\alpha_n + (N-n)\alpha_j}{b} \right) z,$$

and the stability condition

$$\frac{d\dot{z}}{dz} = - \left( k - \frac{n\alpha_n + (N-n)\alpha_j}{b} \right) < 0.$$

Next, using the negative values for  $\alpha_n$  and  $\alpha_j$  we calculate  $\beta_n$  and  $\beta_j$  from (76) and (79) and then  $\mu_n$  and  $\mu_j$  from (77) and (80).

Given these values, if we assume that  $z_0 = 0$ , we have that  $W_i^f = W_n/n = \mu_n/n$  and  $W_j^f = \mu_j$ , so that we are able to write the value functions in terms of  $n$  and analyze the stability of coalitions for the feedback Nash equilibrium.

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<sup>20</sup>We have computed the solution using the MAPLE program.

## 5.4 A numerical example

Our results show, as happened for the open-loop Nash equilibrium, that only a bilateral coalition is stable as long as  $W_i^f(2) > W_j^f(1)$  and  $W_j^f(n-1) > W_i^f(n)$  for any size of coalition higher than two. See Table 4.

**Table 4: Stability analysis for the numerical example. Feedback strategies.\***

n	$q_{i\infty}$	$q_{j\infty}$	$z_{\infty}$	$W_i$	$W_j$	$W$
1		26.114	52227.699		42482548	424825480
2	23.732	26.161	51351.010	42522777	42657598	426306338
3	21.540	26.252	49676.318	42647897	42999455	428939876
4	19.622	26.377	47350.254	42854182	43490779	432361402
5	18.030	26.527	44556.940	43135027	44107150	436210885
6	16.777	26.691	41485.569	43481491	44819793	440168118
7	15.850	26.859	38305.125	43883036	45598495	443976737
8	15.213	27.025	35150.249	44328318	46414217	447454978
9	14.823	27.183	32117.445	44805945	47241061	450494566
10	14.634		29268.293	45305098		453050980

\*Assumes  $k=0.005$ ,  $N=10$ ,  $a=100000$ ,  $b=3500$ ,  $c=0.005$  and  $\delta=0.025$

Again the analysis of sensitivity shows that only a bilateral IEA is self-enforcing independently of the gains coming from cooperation for the fifty cases studied.<sup>21</sup>

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<sup>21</sup>Complete computation of the numerical examples studied in this paper is available upon request.

**Table 5: Parameter values for the sensitivity analysis and reductions in steady-state accumulated emissions (%). Feedback strategies.\***

b	C				
	0.0010	0.0025	0.0050	0.0075	0.0100
1500	27.71	47.44	62.25	69.53	73.88
	17.57	33.98	49.36	58.15	63.84
2500	18.96	36.03	51.52	60.16	65.68
	11.43	24.01	37.92	47.00	53.40
3500	14.40	29.05	43.96	53.04	59.17
	8.47	18.56	30.78	39.44	45.90
4500	11.61	24.34	38.34	47.44	53.84
	6.73	15.13	25.91	33.98	40.25
5500	9.73	20.94	33.99	42.91	49.40
	5.58	12.77	22.37	29.85	35.84

\* Assumes  $k=0.005$ ,  $N=10$  and  $a=100000$ . The top number in each cell represents the gains of cooperation for  $\delta = 0.025$  and the bottom for  $\delta = 0.05$

The unique difference we find with the previous example is that now the percentage gains of cooperation are something greater than when the countries used open-loop strategies. Compare Table 5 with Table 3. This is a consequence of the fact that the noncooperative outcome is less efficient for the feedback Nash equilibrium in linear strategies than for the open-loop Nash equilibrium.<sup>22</sup>

The intuition of this result is that agents do not find it profitable to select punishment strategies as long as they are vulnerable, using the terminology of repeated games theory, to renegotiation. In other words, punishment

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<sup>22</sup>In our example these differences are minimal. This is explained because we impose a lower bound for  $b/c$ , see (67), in order to guarantee that  $q_{i\infty}^{ol}$  is positive for all  $n$ . As a consequence of this we are implicitly defining an upper bound on (51), i.e., an upper bound on the distance, in relative terms, between the steady-state accumulated emissions of the open-loop Nash equilibrium and the steady-state accumulated emissions of the feedback Nash equilibrium that in our example is very small. The result is that for this case the open-loop Nash equilibrium is a good approximation of the feedback Nash equilibrium.

strategies are not credible because they have a negative effect on the own payoffs higher than the negative effect of accommodating to the exit of one of the countries in the agreement. Thus, if we compute the optimal strategies for our example we find that the optimal reaction of a signatory country to an increase of accumulated emissions caused, for instance, by the exit of one country in the agreement, is to reduce emissions and accommodate the exit. In the example of Table 4, for the signatories  $\partial q_i / \partial z$  goes from  $-3.692148(10)^4$  for  $n = 10$  to  $-7.981728(10)^{-5}$  for  $n = 2$ . Then, given that the incentive to act as a free rider is sufficiently large and that the countries in the agreement accommodate the exit, the result is that only a bilateral agreement can be self-enforcing. This minimal level of cooperation appears because the payoffs of the signatories increase with respect to the number of countries belonging to the agreement so that if there are only two countries in the agreement and one of them withdraws from the coalition its payoffs decrease.

A final comment on the feedback Stackelberg equilibrium. For this kind of differential game there is no difference between the Nash equilibrium and the Stackelberg equilibrium. The explanation is very simple. If we look at the necessary conditions (69) and (71), which we can interpret as dynamic reaction functions, we see that the unique interdependence among the countries is through the state variable. In other words, the emissions of country  $i$  do not depend directly on the emissions of country  $j$  so that if we substitute the reaction function of the nonsignatories in the Bellman's equation of signatories, we obtain the same necessary condition for signatories and the system of differential equations (72) and (73) is also the same.

## 6 CONCLUDING REMARKS

This paper has focused on the analysis of stability of voluntary environmental agreements both in a static and in a dynamic framework. Coalition formation has been designed as a noncooperative game where some countries cooperate and sign an agreement and the others do not cooperate. The results show that cooperation improves welfare so that the payoffs of signatory countries monotonically increase as the size of the coalition increases and it is also the case for nonsignatories. However the incentive for a country to act as a free rider is big enough as if to prevent cooperation once a coalition of two countries has been reached. This result is independent of the concept of equilibrium used to solve the game and the framework, static (flow exter-

nality) or dynamic (stock externality), taken into account. For the dynamic model the explanation of this result is that countries do not find it profitable to select punishment strategies because they have a negative effect on their own payoffs higher than the negative effect of accommodating to the exit of one country in the agreement. Then, given that there exists an incentive to act as a free rider, the countries defect from the agreement until this is only constituted by two countries. For two countries this incentive disappears because the two countries lose if they leave the agreement since the payoffs of signatories always increase with respect to the number of countries belonging to the agreement.

As regards the effect of the asymmetry on the scope of the agreement we guess that the result is not going to change very much. The intuition is the following. Suppose that there are two types of countries: small and big. Obviously, big countries could punish small countries without affecting very much their own payoffs but this is not the case if a big country decides to withdraw. Then, if the incentive for a big country to act as a free rider is high enough the country abandons the agreement. On the other hand, an agreement constituted only by small countries is not stable either because again the punishment strategies are not credible. For this reason we expect that the scope of the agreement, even under asymmetry, be limited.<sup>23</sup>

Other issues for future research are the following. A first task could be to consider that there exists imperfect information on benefits and cost functions or uncertainty on environmental damages.<sup>24</sup> Moreover, since emissions, in many cases, are very difficult to monitor, another extension could be the analysis of the profitability and stability of an IEA when the instrument of the environmental policy is a tax on emissions.

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<sup>23</sup>This intuition is supported by the results obtained by Hoel (1992) for a model with heterogeneous countries and constant marginal environmental cost. See the Introduction.

<sup>24</sup>In this line Petrakis and Xepapadeas (1996) have addressed the problem of designing a mechanism to enforce an IEA under moral hazard, and Na and Shin (1998) have analyzed a game of coalition formation among three countries that are not identical when there is uncertainty concerning the distribution of the benefits of pollution abatement activity, and the marginal benefits are linear. Another interesting paper is Batabyal (2000). This author focuses on the problem of designing contracts among a supra-national governmental authority and the government and a representative polluting firm of a developing country when there is uncertainty on the pollution abatement technology.

## A Behavior of emissions with regard to the size of the coalition in the leadership model.

(i) Firstly, we show that emissions of signatories countries decrease with the number of countries that sign the agreement. First, notice that in the numerator of (15),  $[n^2(b+c)c - (b+Nc)^2]$  is an increasing function that equals to zero for  $n^* = (b+Nc)/((b+c)c)^{1/2}$ . Suppose now that  $n^* \leq N$ . This implies that  $b+Nc \leq Nc^{1/2}(b+c)^{1/2}$ . Squaring this expression, simplifying and rearranging terms we get  $b/c < N(N-2)$ , what contradicts the condition established in the Section 2.1 to guarantee positive net benefits for the noncooperative Cournot equilibrium. Therefore, we must conclude that  $n^* > N$  and, as a result, that  $[n^2(b+c)c - (b+Nc)^2]$  is negative so that emissions by signatory countries are decreasing with the number of countries that sign the agreement.

(ii) Secondly, we show that emissions of nonsignatories increase with the number of countries that sign the agreement. In the numerator of (16),  $[n^2(b+c)c - 2n(b+Nc)(b+c) + (b+Nc)^2]$  is a convex function of  $n$  that takes its minimum value for  $n^* = (b+Nc)/c$  and it is equal to zero for

$$\underline{n} = \frac{b+Nc}{c} \left( 1 - \frac{b^{1/2}}{(b+c)^{1/2}} \right), \quad \bar{n} = \frac{b+Nc}{c} \left( 1 + \frac{b^{1/2}}{(b+c)^{1/2}} \right),$$

so that the function takes negative values for  $n \in (\underline{n}, \bar{n})$ . Suppose now that  $\bar{n} \leq N$ , what means that

$$\frac{b+Nc}{c} \left( 1 + \frac{b^{1/2}}{(b+c)^{1/2}} \right) \leq N.$$

>From this inequality we get a contradiction

$$b \left( 1 + \frac{b^{1/2}}{(b+c)^{1/2}} \right) + Nc \frac{b^{1/2}}{(b+c)^{1/2}} \leq 0,$$

so that we can conclude that  $N < \bar{n}$ . Suppose now that  $\underline{n} \geq 1$ , this yields that

$$\frac{b+Nc}{c} \left( 1 - \frac{b^{1/2}}{(b+c)^{1/2}} \right) \geq 1.$$



Rearranging terms we obtain

$$\frac{1}{(b+c)^{1/2}c} \left[ (b+c)^{1/2} (b+(N-1)c) - (b+Nc) b^{1/2} \right] \geq 0,$$

what implies that

$$(b+c)^{1/2} (b+(N-1)c) \geq (b+Nc) b^{1/2},$$

and squaring this expression we get

$$(N-1)^2 c^2 \geq (b+c) b. \quad (81)$$

On the other hand, we have assumed in Section 2.1 that  $b \geq N(N-2)c$ . Multiplying this inequality by  $b+c$  we have that  $(b+c)b \geq (N-2)Nc(b+c)$  that in combination with (81) imposes the following condition on  $b : c/(N-2)N > b$ , which is not compatible with the previous condition on  $b$  since both conditions imply that  $1 > (N-2)^2 N^2$ , which is not true for  $N \geq 3$ . Therefore, we can conclude that the function of  $n$  in the denominator of (16) takes negative values for  $n \in (\underline{n}, \bar{n})$ , with  $0 \leq \underline{n} \leq 1 < N < \bar{n}$ , so that for  $n \in [2, N]$ , emissions by nonsignatory countries increase with the size of the coalition.

## B Discounted present value for the cooperative solution

Substituting in the expression for the discounted present value of net benefits we obtain

$$\begin{aligned} W_i^* = & \int_0^\infty e^{-\delta t} \left[ a q_{i\infty}^* - \frac{Nca}{b(r^* - \delta - k)} z_\infty^* e^{r^* t} \right. \\ & - \frac{b}{2} \left( q_{i\infty}^* - \frac{Nc}{b(r^* - \delta - k)} z_\infty^* e^{r^* t} \right)^2 \\ & \left. - \frac{c}{2} \left( z_\infty^* (1 - e^{r^* t}) \right)^2 \right] dt. \end{aligned}$$

Squaring and rearranging terms we have

$$W_i^* = \pi_{i\infty}^* \int_0^\infty e^{-\delta t} dt - c I z_\infty^* \int_0^\infty e^{(r^* - \delta)t} dt - \frac{c}{2} J (z_\infty^*)^2 \int_0^\infty e^{(2r^* - \delta)t} dt,$$

where

$$I = \frac{N(a - bq_{i\infty}^*)}{b(r^* - \delta - k)} - z_\infty^*, \text{ and } J = 1 + \frac{N^2 c}{b(r^* - \delta - k)^2}.$$

Integrating we get

$$W_i^* = \frac{\pi_{i\infty}^*}{\delta} + \frac{c}{r^* - \delta} I z_\infty^* + \frac{c}{2(2r^* - \delta)} J (z_\infty^*)^2.$$

Now, if we take into account that

$$a - bq_{i\infty}^* = -\lambda_{i\infty}^* = \frac{cNz_\infty^*}{\delta + k},$$

we can write  $I$  as

$$I = z_\infty^* \left( \frac{cN^2 - b(r^* - \delta - k)(\delta + k)}{b(r^* - \delta - k)(\delta + k)} \right),$$

and, then, rearranging terms we obtain

$$W_i^* = \frac{\pi_{i\infty}^*}{\delta} + \frac{c}{b(r^* - \delta - k)} K (z_\infty^*)^2$$

with

$$K = \frac{cN^2 - b(r^* - \delta - k)(\delta + k)}{(r^* - \delta)(\delta + k)} + \frac{cN^2 + b(r^* - \delta - k)^2}{2(2r^* - \delta)(r^* - \delta - k)}.$$

## C Comparison of the two noncooperative solutions

First, we compare the steady-state values for accumulated emissions. Using (31) and (47), we find that the difference between the two steady-state values can be written as

$$z_{\infty}^{ol} - z_{\infty}^f = \frac{aN^2 \{(\delta + k) [(2N - 1) \alpha_i^2 - 2b(k + \frac{\delta}{2}) \alpha_i - bc] + (N - 1) c \alpha_i\}}{b[k((\delta + k)b - (N - 1) \alpha_i) + Nc][k(\delta + k)b + Nc]},$$

where the denominator is positive as long as  $\alpha_i$  is negative. Then, taking account that  $(2N - 1) \alpha_i^2 - 2b(k + \frac{\delta}{2}) \alpha_i - bc = 0$ , according to (39), we obtain a negative difference for the numerator and we can conclude that  $z_{\infty}^{ol}$  is lower than  $z_{\infty}^f$ . To compare the steady-state emissions we take into consideration that  $q_{i\infty} = (k/N) z_{\infty}$  for the two solutions so that we can write the following difference

$$q_{i\infty}^{ol} - q_{i\infty}^f = \frac{k}{N} (z_{\infty}^{ol} - z_{\infty}^f)$$

which has a negative sign.

Now, in order to compare the optimal paths we define  $r^f$  as  $-k + (N\alpha_i)/b$  and we compare it with the negative root of the open-loop Nash equilibrium,  $r^{ol}$ . Let's assume that  $|r^f| \geq |r^{ol}|$ , then using (42) to eliminate  $\alpha_i$  we obtain the following inequality

$$\begin{aligned} & \frac{2(N-1)}{2N-1} \left(k + \frac{\delta}{2}\right) + \frac{2N}{2N-1} \sqrt{\left(k + \frac{\delta}{2}\right)^2 + \frac{2N-1}{b}c} \\ & \geq \sqrt{\delta^2 + \frac{4}{b}[k(\delta+k)b + Nc]}. \end{aligned}$$

Squaring and simplifying terms, we obtain that

$$2 \left(k + \frac{\delta}{2}\right) \sqrt{\left(k + \frac{\delta}{2}\right)^2 + \frac{2N-1}{b}c} \geq 2 \left(k + \frac{\delta}{2}\right)^2 + \frac{2N-1}{b}c,$$

and squaring again and simplifying terms, we finally obtain the following contradiction

$$0 \geq \frac{(2N-1)^2}{b^2} c^2,$$

and we can conclude that  $|r^f| < |r^{ol}|$ . Now, from (32) and (49) we obtain that

$$\dot{z}^{ol} = \frac{dz^{ol}}{dt} = z_{\infty}^{ol} |r^{ol}| e^{r^{ol}t} \text{ and } \dot{z}^f = \frac{dz^f}{dt} = z_{\infty}^f |r^f| e^{r^f t}$$

where  $|r^f| < |r^{ol}|$ . The rest of the proof follows the proof of Proposition 8 step by step. For this reason, this part has been omitted.

## D Percentage variation of steady-state accumulated emissions

We can rewrite (50) as

$$\Delta z = \frac{z_{\infty}^* - z_{\infty}^f}{z_{\infty}^f} = -N(N-1)F,$$

where

$$F = \frac{(\delta + k - N(\alpha_i/b))}{(k(\delta + k)(b/c) + N^2)(\delta + k - (N-1)(\alpha_i/b))}.$$

The derivative of the numerator is  $-N \frac{\partial(\alpha_i/b)}{\partial(b/c)}$ , being  $\frac{\partial(\alpha_i/b)}{\partial(b/c)} = \frac{1}{2(b/c)^2} G^{-1/2} > 0$ ,  $G = \left(k + \frac{\delta}{2}\right)^2 + \frac{2N-1}{(b/c)}$ .

The derivative of the denominator is  $k(\delta + k)(\delta + k - (N-1)(\alpha_i/b)) - (k(\delta + k)(b/c) + N^2)((N-1)/2(b/c)^2)G^{-1/2}$  whose sign is ambiguous. Using these two derivatives we can calculate the numerator of  $\partial F / \partial(b/c)$  :

$$\begin{aligned} & -\frac{N}{2(b/c)^2} G^{-1/2} (k(\delta + k)(b/c) + N^2)(\delta + k - (N-1)(\alpha_i/b)) \\ & -(\delta + k - N(\alpha_i/b)) [k(\delta + k)(\delta + k - (N-1)(\alpha_i/b)) \\ & - (k(\delta + k)(b/c) + N^2) \frac{N-1}{2(b/c)^2} G^{-1/2}], \end{aligned}$$

taking common factor and simplifying we obtain a negative value<sup>25</sup>

$$-(\delta + k) \left[ \frac{G^{-1/2}}{2(b/c)^2} (k(\delta + k)(b/c) + N^2) \right. \\ \left. + k(\delta + k - N(\alpha_i/b))(\delta + k - (N - 1)(\alpha_i/b)) \right],$$

so that  $\partial F / \partial (b/c)$  is negative and consequently  $\partial \Delta z / \partial (b/c)$  is positive.

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<sup>25</sup>Remember that  $\alpha_i/b$  is negative.

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